

Supergravity interacting with superbranes and spacetime Higgs effect in General Relativity ¹

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Abstract

In this contribution we will review briefly the supersymmetric Lagrangian approach to the supergravity–superbrane interaction which was developed in collaboration with J. A. de Azcárraga, J.M. Izquierdo, J. Lukierski and J. M. Isidro. The main accent will be made on the pure gauge nature of the (super)brane coordinate functions in the presence of dynamical (super)gravity described by an action rather than as a fixed background. This pure gauge nature just reflects the fact that the coordinate functions are Goldstone fields corresponding to the spontaneously broken diffeomorphism gauge symmetry of the interacting system. Moreover, a brane does not carry any local degrees of freedom in such an interacting system. This fact related with fundamental properties of General Relativity (discussed already at 1916) can be treated as a peculiarity of the spacetime Higgs effect which occurs in General Relativity in the presence of material particles, strings and branes.

1. Introduction. Let me begin by thanking the organizers for the nice opportunity to speak at this conference dedicated to José Adolfo de Azcarraga. I had already known him through his papers for more than 20 years. At that time, working in Kharkov in the group of Dmitri Volkov, we had been especially influenced by the work [1], where κ -symmetry was found, and later by the topological treatment of tensorial ‘central’ charges of the most general supersymmetry algebra [2] (see the contribution of D. Sorokin [3] for more details). In the last few years I had the pleasure to know José Adolfo personally, to enjoy his friendship and to collaborate with him. I have enjoyed very much discussing with José, and not only on physics: he has widespread interests in many other areas, both scientific and non-scientific.

Our collaboration has been not only pleasant but quite productive [4, 5, 6, 7, 8, 9]. Interestingly enough, the field of our studies can be separated into two streams, one [4] being related to the work [2] (as well as with more recent studies of [10]) and the other [5, 6, 7, 8, 9], dealing with the Lagrangian description of dynamical supergravity interacting with supersymmetric extended objects, is related to the κ -symmetry of superbrane (see below), and thus, with the work of [1]. This latter direction will be the main subject of the present contribution.

The results of [6, 7, 8] have a clear ‘bosonic projection’ which appeared to be related [9] with fundamental properties of General Relativity (quite well known and actively discussed decades ago [11, 12, 13]). In the modern language, they may be treated as an analysis of a spacetime counterpart of Higgs effect

¹To appear in “*Symmetries in gravity and field theory*. Conferencia homenaje en el 60 cumpleaños de José Adolfo de Azcárraga. Salamanca. June 9-11, 2003”.

which occurs in general relativity in the presence of material particles, strings or p -branes.

2. Problem of supersymmetric Lagrangian description of supergravity–superbrane interaction. A popular description of superbranes was proposed in 1989 [14, 15]. It identified them with solitonic solutions of the pure bosonic ‘limit’ of the supergravity equations. Although all the fermions are set equal to zero, the superbrane solutions are supersymmetric and, hence, stable.

On the other hand, one may also consider a superstring or superbrane in a curved superspace defined by a supergravity *background* [16, 17]. Then, self-consistency requires having a smooth flat superspace limit for such a system. This implies, in particular, that the superbrane has to possess local fermionic κ -symmetry [1, 18] in curved superspace, as it does in the flat one. Such a requirement immediately results in superfield supergravity constraints being imposed on the background superfields [16, 17]. The point is that, for the most interesting (in M-theoretical perspective) $D = 10, 11$ cases these are the *on-shell* constraints: their selfconsistency implies equations of motion for $D = 10$ supergravity, and these are sourceless or ‘free’ equations. Clearly, such a description is an approximate one.

In purely bosonic ‘limit’ one can describe the interacting system of dynamical gravity and a p -dimensional material object (p -brane) by the action

$$S = S_{EH D} + S_{p D} = \frac{1}{2\kappa} \int d^D x \sqrt{|g|} \mathcal{R} + S_{p D} , \quad (1)$$

$$S_{p D} = \frac{T_p}{4} \int d^{p+1} \xi [\sqrt{|\gamma|} \gamma^{mn}(\xi) \partial_m \hat{x}^\mu \partial_n \hat{x}^\nu g_{\mu\nu}(\hat{x}) + (p-1) \sqrt{|\gamma|}] . \quad (2)$$

which is the sum of Einstein–Hilbert action for gravity $S_{EH D}$ and the bosonic p -brane action $S_{p D}$ written in terms of coordinate functions $\hat{x}^\nu(\xi)$ determining the position of the p -brane worldvolume W^{p+1} in the spacetime [$\xi^m = (\tau, \sigma^1, \dots, \sigma^p)$ are local coordinates on W^{p+1} , $\gamma_{mn}(\xi)$ is an auxiliary worldvolume metric; on the mass shell it coincides with the induced metric, $\gamma_{mn}(\xi) = \partial_m \hat{x}^\mu \partial_n \hat{x}^\nu g_{\mu\nu}(\hat{x})$]. The variation of the action with respect to the metric $g_{\mu\nu}(x)$ (see [15]) produces the Einstein equation

$$\mathcal{G}_{\mu\nu} \equiv \sqrt{|g|} (\mathcal{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \mathcal{R}) = \kappa T_{\mu\nu} \quad (3)$$

with the singular energy–momentum tensor from the p -brane

$$T^{\mu\nu} = \frac{T_p}{4} \int d^{p+1} \xi \sqrt{\gamma} \gamma^{mn} \partial_m \hat{x}^\mu \partial_n \hat{x}^\nu \delta^D(x - \hat{x}(\xi)) , \quad (4)$$

while the variation with respect to coordinate functions $\hat{x}^\mu(\xi)$ produces the equations of motion for the p -brane

$$\partial_m (\sqrt{|\gamma|} \gamma^{mn} g_{\mu\nu}(\hat{x}) \partial_n \hat{x}^\nu(\xi)) - \frac{1}{2} \sqrt{|\gamma|} \gamma^{mn} \partial_m \hat{x}^\nu \partial_n \hat{x}^\rho (\partial_\mu g_{\nu\rho})(\hat{x}) = 0 . \quad (5)$$

Is it possible to provide a full (quasi)classical Lagrangian description of the supergravity–superbrane interacting system similar to the description of the

gravity–bosonic brane interaction given by the action (1)? It was natural to assume that it is based on the sum

$$S = S_{SG} + S_p^s$$

of *some* supergravity action S_{SG} and the super- p –brane action S_p^s , the same as in the case of supergravity background.

A superbrane is a brane moving in superspace $Z^M = (x^\mu, \theta^{\hat{\alpha}})$, *i.e.* its worldvolume \mathcal{W}^{p+1} is a surface in superspace which can be defined by a set of bosonic and fermionic coordinate functions $\hat{Z}^M(\xi^m) = (\hat{x}^\mu(\xi), \hat{\theta}^{\hat{\alpha}}(\xi))$,

$$\mathcal{W}^{p+1} \subset \Sigma^{(D|n)} : Z^M = \hat{Z}^M(\xi^m) := (\hat{x}^\mu(\xi), \hat{\theta}^{\hat{\alpha}}(\xi)), \quad m = 0, \dots, p, \quad (6)$$

The action S_p^s of a super- p –brane in a curved superspace [16, 17, 19] is formulated in terms of supergravity superfields, supervielbeins $E^A(Z) = dZ^M E_M^A(Z) = (E^a, E^\alpha)$ and superform gauge superfields $C_q(Z) = \frac{1}{q!} dZ^{M_q} \wedge \dots \wedge dZ^{M_1} C_{M_1 \dots M_q}(Z)$. For instance, for the super- p –branes of the ‘old brane scan’ [19] one has $S_p^s = 1/4 * \hat{E}_a \wedge \hat{E}^a - (-)^p(p-1)/4 * 1 - \hat{C}_{p+1}$, or, equivalently,

$$\begin{aligned} S_p^s = & \int_{\mathcal{W}^{p+1}} d^{p+1}\xi \sqrt{|\gamma|} \left[\gamma^{mn}(\xi) \partial_n \hat{Z}^N \partial_m \hat{Z}^M E_M^a(\hat{Z}) E_N^b(\hat{Z}) \eta_{ab} + (p-1) \right] - \\ & - \int_{\mathcal{W}^{p+1}} \frac{1}{p!} \varepsilon^{m_1 \dots m_{p+1}} \partial_{m_{p+1}} \hat{Z}^{M_{p+1}} \dots \partial_{m_1} \hat{Z}^{M_1} C_{M_1 \dots M_{p+1}}(\hat{Z}). \end{aligned} \quad (7)$$

Note that the superbrane action involves *only bosonic superforms*; the fermionic supervielbein $E^\alpha = dZ^M E_M^\alpha(Z)$ is not present in $S_p = S_p[\hat{E}^a, \hat{C}_{p+1}]$. Here and below the hats over superfields or superforms indicate dependence on the coordinate functions *i.e.*, $\hat{Z}^M(\xi) = (\hat{x}^\mu(\xi), \hat{\theta}^{\hat{\alpha}}(\xi))$ replace the superspace coordinates $Z^M = (x^\mu, \theta^{\hat{\alpha}})$. The hat also distinguishes between the coordinate functions and the spacetime or superspace coordinates.

To be able to derive the equation of motion, one should have supergravity part S_{SG} of the action $S = S_{SG} + S_p[\hat{E}^a, \hat{C}_q]$ formulated in terms of the same variables; *i.e.* one should use the *superfield supergravity action* $S_{SG}[E^A, C_q]$,

$$S = S_{SG}[E^A, C_q] + S_p[\hat{E}^a, \hat{C}_q] \equiv S_{SG}[E^A(Z), C_q(Z)] + S_p^s[E^a(\hat{Z}), C_q(\hat{Z})], \quad (8)$$

In the $D = 4$, $N = 1$ case the superfield action for supergravity is known [$S_{SG}[\hat{E}^A] = \int d^8 Z s \det(E_M^A(Z))$] and one can study the interacting system (8). This has been done for supergravity–superparticle and supergravity–superstring interacting systems [7, 20]. The problem, however, is that for the most interesting $D = 10$ and $D = 11$ cases no *superfield* action for supergravity is known.²

²As, in contrast, the *component* action $S_{SG}[e^a, dx^\mu \psi_\mu^\alpha, \dots]$ for all $D \leq 11$ supergravity theories is now known, one might think of using it in (8) instead of the superfield supergravity action, and decompose the superfields in $S_p[\hat{E}^a, \hat{C}_q]$ in terms of component fields, *e.g.* $E_\mu^a = e_\mu^a(x) + \mathcal{O}(\theta)$, $E_\mu^\alpha = \psi_\mu^\alpha(x) + \mathcal{O}(\theta)$. However, despite the first impression the variational problem for the action $S_{SG}[e^a, dx^\mu \psi_\mu^\alpha, \dots] + S_p[\hat{E}^a, \hat{C}_q]$ is not well posed, see [8].

3. A gauge-fixed description of dynamical supergravity interacting with material superparticles, superstrings and superbranes

3.1. *A practical way to study the supergravity-superbrane interacting system* without a use of the superfield supergravity action was proposed in [7, 8]. It was shown there that the dynamical system described by the superfield action (8) (still hypothetical and, possibly, non existing in $D = 10, 11$) is gauge equivalent to the more simple dynamical system described by the sum

$$S = S_{SG}[e^a(x), dx^\mu \psi_\mu^\alpha(x), C_q(x)] + S_p[\hat{e}^a, \hat{C}_q] , \quad (9)$$

of the spacetime (component) action for supergravity $S_{SG}[e^a(x), \psi^\alpha(x), C_q(x)]$ *without auxiliary fields* and the action for the bosonic brane, $S_p[\hat{e}^a, \hat{C}_q(\hat{x})]$, which is given by a purely bosonic 'limit' of the superbrane action $S_p[\hat{e}^a, \hat{C}_q] \equiv S_p^s[\hat{E}^a, \hat{C}_q(\hat{Z})]|_{\hat{\theta}=0}$. Such a gauge fixed description is complete in the sense that it produces the gauge fixed version of all the dynamical equations of the complete interacting system [6, 7]. It also possesses the 1/2 of the local supersymmetry characteristic of the 'free' supergravity action $S_{SG}[e^a, \psi^\alpha, C_q(x)]$ [6]. This preserved 1/2 of the supersymmetry reflects the κ -symmetry of the prototype superbrane action [6].

3.2. *Pure gauge nature of the superbrane coordinate functions in the presence of dynamical supergravity.* What is stated above implies that the fermionic coordinate functions of a superbrane, $\hat{\theta}^{\tilde{\alpha}}(\xi)$ (not to be confused with the superspace coordinates $\theta^{\tilde{\alpha}}$), *have a pure gauge nature*. The gauge symmetry which can be used to gauge it away

$$\hat{\theta}^{\tilde{\alpha}}(\xi) = 0 . \quad (10)$$

is the superdiffeomorphism symmetry or *passive form of the superspace general coordinate invariance*. This implies arbitrary, but invertible, transformations of the superspace coordinates

$$\delta_{sdiff} x^\mu = b^\mu(x, \theta) = b^\mu(x) + \mathcal{O}(\theta) , \quad (11)$$

$$\delta_{sdiff} \theta^{\tilde{\alpha}} = b^{\tilde{\alpha}}(x, \theta) = \varepsilon^{\tilde{\alpha}}(x) + \mathcal{O}(\theta) , \quad (12)$$

but leaves invariant differential forms, *e.g.* $E^{A'}(Z') = E^A(Z)$, where $Z^{M'} = Z^M + \delta_{sdiff} Z^M = Z^M + b^M(Z)$. The leading terms $b^\mu(x) = b^\mu(x, 0)$ and $\varepsilon^{\tilde{\alpha}}(x) = b^{\tilde{\alpha}}(x, 0) = b^{\tilde{\alpha}}|_{\theta=0}$ in the decomposition of the superdiffeomorphism parameters $b^M(Z) = (b^\mu(Z), b^{\tilde{\alpha}}(Z))$ can be identified with the parameters of spacetime diffeomorphisms ($b^\mu(x)$) and of the *local supersymmetry* ($\varepsilon^{\tilde{\alpha}}(x)$), of the *component* (spacetime) formulation of the supergravity.

In the supergravity-superbrane interacting system described by the action (8) the superdiffeomorphisms act also on the coordinate function by $\delta_{sdiff} \hat{x}^\mu(\xi) = b^\mu(\hat{x}(\xi), \hat{\theta}(\xi))$, $\delta_{sdiff} \hat{\theta}^{\tilde{\alpha}}(\xi) = b^{\tilde{\alpha}}(\hat{x}(\xi), \hat{\theta}(\xi))$. The interacting system also possesses *reparametrization* symmetry (or worldvolume diffeomorphism invariance) [$\delta_{rep} \xi^m = \beta^m(\xi)$, $\hat{Z}'^M(\xi + \delta_{rep} \xi) = \hat{Z}^M(\xi)$] and κ -symmetry [$\delta_\kappa \hat{Z}^M = \kappa^\alpha (1 - \bar{\Gamma})_\alpha{}^\beta E_\beta{}^M(\hat{Z})$ with a certain projector $(1 - \bar{\Gamma})_\alpha{}^\beta$ ($tr \bar{\Gamma} = 0$, $\bar{\Gamma} \bar{\Gamma} = I$)], both acting

on the coordinate functions only. Thus the complete set of gauge transformations acting on the coordinate functions read

$$\delta_{gauge}\hat{x}^\mu(\xi) = b^\mu(\hat{x}(\xi), \hat{\theta}(\xi)) + \delta_\kappa\hat{x}^\mu(\xi) + \delta_{rep}\hat{x}^\mu(\xi) \quad (13)$$

$$\delta_{gauge}\hat{\theta}^{\hat{\alpha}}(\xi) = b^{\hat{\alpha}}(\hat{x}(\xi), \hat{\theta}(\xi)) + \delta_\kappa\hat{\theta}^{\hat{\alpha}}(\xi) + \delta_{rep}\hat{\theta}^{\hat{\alpha}}(\xi) . \quad (14)$$

The gauge (10) can be fixed using the fermionic superdiffeomorphisms (actually its leading component, the local supersymmetry $\varepsilon^{\hat{\alpha}}(x)$). Then the symmetry preserving the gauge (10) is defined by $\delta_{gauge}\hat{\theta}^{\hat{\alpha}}(\xi)|_{\hat{\theta}=0} = 0$, which (in the light of $\delta_{rep}\hat{\theta}^{\hat{\alpha}}(\xi)|_{\hat{\theta}=0} \equiv 0$) implies

$$\delta_{gauge}\hat{\theta}^{\hat{\alpha}}(\xi)|_{\hat{\theta}=0} = 0 \quad \Rightarrow \quad \varepsilon^{\hat{\alpha}}(\hat{x}) = -\delta_\kappa\hat{\theta}^{\hat{\alpha}}(\xi)|_{\hat{\theta}=0} \equiv -\kappa^\alpha(1 - \bar{\gamma})_\alpha{}^\beta\delta_\beta{}^{\hat{\alpha}} . \quad (15)$$

This shows the preservation of the 1/2 of the local supersymmetry on the world-volume as well as the relation of this preserved supersymmetry with the κ -symmetry of the parent superbrane action [$\bar{\gamma} = \bar{\Gamma}|_{\hat{\theta}=0}$ is the ‘leading component’ of the parent κ -symmetry projector, see [6]].

3.3. How to arrive at the gauge fixed action (9).

An important property of the gauge (10) is that it can be fixed simultaneously with the Wess–Zumino (WZ) gauge for supergravity ³. In the WZ gauge one can perform the Grassmann integration in the superfield supergravity action S_{SG} and arrive at a spacetime (component) action for supergravity, but with auxiliary fields, $S_{SG}[e^a(x), \psi^\alpha(x), C_q(x), aux.fields]$. Then one has to observe that *i*) the leading components of all superfields/superforms of the superfield supergravity are certain physical fields of the supergravity multiplet, *e.g.* $E^a|_{\theta=0, d\theta=0} = dx^\mu e_\mu^a(x)$, $E^\alpha|_{\theta=0, d\theta=0} = dx^\mu \psi_\mu^\alpha(x)$, *ii*) all the super- p -brane actions S_p^s (including the ones from the ‘old brane scan’ [17, 19], Eq. (7) as well as Dirichlet superbranes and M5-brane) involve only bosonic superfields of the supergravity, $S_p^s = S_p[E^a(\hat{Z}), C_q(\hat{Z})]$. Thus in the gauge (10) the superbrane action S_p^s becomes the action for a bosonic brane $S_p[e^a(\hat{x}), C_q(\hat{x})] = S_p^s[E^a(\hat{Z}), C_q(\hat{Z})]|_{\hat{\theta}=0}$ involving *only physical bosonic fields of the supergravity multiplet*. No auxiliary fields enter in this bosonic brane action. As so, the auxiliary fields can be removed from $S_{SG}^0(e^a(x), \psi^\alpha(x), C_q(x), aux.fields)$ by using their purely algebraic equations of motion, in the same manner as it is done in the case of ‘free’ supergravity.

After this stage one arrives at the gauge fixed action of the form (9).

The gauge fixed description of the supergravity—superbrane interacting system provided by this action is quite practical because *i*) it involves only the component action for supergravity known in all dimensions including $D = 10, 11$, *ii*) it provides a gauge fixed description of the supergravity–superbrane interacting system which is *complete* in the sense that it produces the gauge fixed versions of all the dynamical equations following from the original superfield action (8)

³See [7] and refs therein; the WZ gauge implies, in particular, $E_\beta{}^{\hat{\alpha}}(x, 0) = \delta_\beta{}^{\hat{\alpha}}$ which has been already used in the second form of Eq. (15).

[6, 7] [the reasons for this may be understood from the discussion below]; *iii*) it still possesses 1/2 of the local supersymmetry of the ‘free’ supergravity action; moreover, this ‘preserved’ 1/2 of the local supersymmetry, on one hand, reminisces the κ -symmetry of the prototype super- p -brane action and, on the other hand, is related with supersymmetry preserved by the super- p -brane BPS state as described by (pure bosonic) solitonic solution of supergravity.

4. The fate of the brane degrees of freedom in General Relativity interacting with material particles, strings and branes.

The above consideration on the pure gauge nature of the fermionic coordinate functions of a superbrane has a clear bosonic counterpart. The bosonic (super)diffeomorphisms act additively on the bosonic coordinate function $\hat{x}^\mu(\xi)$, Eq. (13). Certainly, in contrast with fermionic case, one cannot set all the coordinate functions to zero [as diffeomorphisms are invertible transformations and, hence, cannot map a (region of a) surface into a point⁴]. However, one can use the diffeomorphisms to fix *locally* (in a tubular neighborhood of a point of \mathcal{W}^{p+1}) the *static gauge* [9]

$$\hat{x}^\mu = (\xi^m, \vec{0}) = (\tau, \sigma^1, \dots, \sigma^p, 0, \dots, 0) , \quad (16)$$

where first $(p+1)$ coordinate functions are identified with the local worldvolume coordinates, $\hat{x}^m(\xi) = \xi^m$, while the remaining $(D - p - 1)$ coordinate functions vanish, $\hat{x}^I(\xi) = 0$.

All this is true also in pure bosonic case, which corresponds to General Relativity interacting with material particles, strings or branes described by the action (1), (2). Thus, this property should be well known in general relativity, at least for the $D = 4$, $p = 0$ case corresponding to General Relativity interacting with a material particle

$$S = S_{EH} + S_{0m} := S_{EH} + \frac{1}{2} \int d\tau (l(\tau) g_{\mu\nu}(\hat{x}) \dot{\hat{x}}^\mu(\tau) \dot{\hat{x}}^\nu(\tau) + l^{-1}(\tau) m^2) . \quad (17)$$

This is indeed the case [9]: the pure gauge nature of the particle coordinate function was known already in 1926 [12], but, clearly, was discussed using different terminology. It is well known that the Bianchi identity $\mathcal{G}^\mu_{\nu;\mu} \equiv 0$ for the Einstein tensor *density* $\mathcal{G}^{\mu\nu}$ in the gravitational field equations (3) implies the covariant conservation of the energy-momentum tensor density

$$T^\mu_{\nu;\mu} \equiv \partial_\mu (T^{\mu\rho} g_{\rho\nu}) - \frac{1}{2} T^{\mu\rho} \partial_\nu g_{\rho\mu} = 0 . \quad (18)$$

For a particle $T_{\mu\nu}$ has support on the worldline \mathcal{W}^1 ,

$$T^{\mu\nu} = \frac{1}{2} \int d\tau l(\tau) \dot{\hat{x}}^\mu(\tau) \dot{\hat{x}}^\nu(\tau) \delta^4(x - \hat{x}(\tau)) , \quad (19)$$

and then Eq. (18) is equivalent [12] to the particle (geodesic) equations

$$\mathcal{S}_{\tau\mu} := \partial_\tau (l(\tau) g_{\mu\nu}(\hat{x}) \dot{\hat{x}}^\nu) - \frac{l(\tau)}{2} \dot{\hat{x}}^\nu \dot{\hat{x}}^\rho (\partial_\mu g_{\nu\rho})(\hat{x}) = 0 \quad (20)$$

⁴In contrast, the gauge $\hat{\theta}^{\tilde{\alpha}}(\xi) = 0$ is fixed by using $b^{\tilde{\alpha}}(\hat{x}, 0)$ parameter, and this does not enter in the matrix $\partial b^{\tilde{\alpha}}(x, \theta) / \partial \theta^{\tilde{\gamma}}$ which has to be nondegenerate to provide invertibility of the superdiffeomorphism transformations.

or $d^2\hat{x}^\mu/ds^2 + \Gamma_{\nu\rho}^\mu d\hat{x}^\nu/ds d\hat{x}^\rho/ds = 0$ for $ds = d\tau/l(\tau)$. The counterparts of the above statement for the cases of gravity interacting with strings and p -branes were given in [21] and [6].

This result exhibits a dependence among Eqs. (20) and (3) which, by the second Noether theorem, reflects a gauge symmetry of the action (17) leading to Eqs. (3) and (20) when varied with respect to the metric $g_{\mu\nu}$ and with respect to the particle coordinate functions \hat{x}^μ , respectively. This gauge symmetry is just the *diffeomorphism invariance* (the freedom of choosing a *local coordinate system*) or *passive* form of the general coordinate invariance (*cf.* Eq. (11))

$$\delta x^\mu \equiv x^{\mu'} - x^\mu = b^\mu(x) , \quad (21)$$

$$\begin{aligned} \delta' g_{\mu\nu}(x) \equiv g'_{\mu\nu}(x) - g_{\mu\nu}(x) &= -(b_{\mu;\nu} + b_{\nu;\mu}) \\ &\equiv -(\partial_\mu b^\rho g_{\nu\rho} + \partial_\nu b^\rho g_{\mu\rho} + b^\rho \partial_\rho g_{\mu\nu}) , \end{aligned} \quad (22)$$

$$\delta \hat{x}^\mu(\tau) \equiv \hat{x}^{\mu'}(\tau) - \hat{x}^\mu(\tau) = b^\mu(\hat{x}(\tau)) . \quad (23)$$

Indeed, the general variation of the action (24) reads

$$\delta S = -\frac{1}{2\kappa} \int d^4x \mathcal{G}^{\mu\nu} \delta' g_{\mu\nu}(x) + \frac{1}{2} \int d^4x T^{\mu\nu} \delta' g_{\mu\nu}(x) - \int d\tau \mathcal{S}_{\tau\mu}(\tau) \delta \hat{x}^\mu(\tau) , \quad (24)$$

where $\mathcal{G}_{\mu\nu}$, $T_{\mu\nu}$ and $\mathcal{S}_{\tau\mu}$ are defined in Eqs. (3), (19) and (20), respectively. Substituting (22), (23), one finds that the first term vanishes since $\mathcal{G}^\mu{}_{\nu;\mu} \equiv 0$, and that the second and third terms cancel (using the second form of Eq. (22)).

Note, that when ‘proved the second Noether theorem’ for diffeomorphisms, in (24) we did not use the the variation δx^μ (21), which is nonvanishing for the diffeomorphism transformations. We are allowed to do this because this variation, when considered without the additional ones (22), (23),

$$\delta x^\mu \equiv x^{\mu'} - x^\mu = t^\mu(x) , \quad (25)$$

$$\delta' g_{\mu\nu}(x) \equiv g'_{\mu\nu}(x) - g_{\mu\nu}(x) = 0 , \quad \delta \hat{x}^\mu(\xi) = 0 , \quad (26)$$

also leaves invariant the Einstein action (and does not act on the particle action). This is the *active form* of general coordinate invariance (see [11] and [6, 13]).

Actually, one can easily prove (see [6]) that *any action invariant under diffeomorphism gauge symmetry is automatically gauge invariant under the active form of general coordinate symmetry* (25), (26). This fact had been known and led to a conceptual discussion on the lack of physical meaning of the space-time point notion in general relativity [11] (see also [13]). The passive form of the general coordinate invariance (diffeomorphism symmetry), Eqs. (21), (22), (23), was desirable as it provided a realization of the general relativity principle. However, the fact that the Einstein–Hilbert action and the Einstein equations are invariant as well under the *active* form of the general coordinate transformations, *i.e.* under a change of ‘physical’ spacetime points, Eqs. (25), (26), “takes away from space and time the last remnant of physical objectivity” [11, 13].

This gauge symmetry also implies that the *material particle, string or brane do not carry any local degrees of freedom in the presence of dynamical gravity*,

described by the action functional, and not given as a fixed background⁵. Indeed, the local brane degrees of freedom could have a meaning by specifying its position in spacetime M^D , i.e. by relating a point/region of \mathcal{W}^{p+1} with a point/set of points in M^D . However, as in a general coordinate invariant theory the spacetime point notion becomes ‘unphysical’ [it is not a gauge invariant concept and thus *cannot* be treated as an observable in so far as observables are identified with gauge invariant entities], the only physically significant information is the existence of the brane worldvolume and, if there are several branes, also the possible intersections of their worldvolumes⁶. This implies, and it is implied by, the pure gauge nature of the *local* degrees of freedom of a brane interacting with dynamical gravity.

The above discussion restores the ‘symmetry’ between the bosonic and the fermionic degrees of freedom of superbrane. Indeed, in the gauge-fixed description of the supergravity—superbrane interaction, Eq. (9), there are no traces of the fermionic degrees of freedom of the superbranes. Then the discussion above shows that in this interacting systems the superbrane does not carry any *local* bosonic degrees of freedom either. The global, topological degrees of freedom are of course still present [a closed brane differs from an open one and, if the dynamical system involves two or more particles or branes, intersecting branes are different from the nonintersecting ones] and have no fermionic counterparts. This, however, is natural in so far the topology of Grassmann algebra is trivial (‘Ectoplasm has no topology’, J.S. Gates).

5. The Higgs effect in General Relativity interacting with material particles and extended objects.

The pure gauge nature of the (super)brane coordinate function in (super)gravity—(super)brane interacting system is actually not surprising. Indeed, it is well known (see [22]) that, for a brane in flat spacetime, the coordinate functions are essentially Goldstone fields corresponding to the generators of rigid translational symmetry broken by the presence of the p -brane. When the brane interacts with *dynamical* gravity, the rigid translational symmetry is replaced by the *gauge* diffeomorphism symmetry. Thus the coordinate functions $\hat{x}^\mu(\tau)$ (and $\theta^{\hat{\alpha}}(\tau)$) become the Goldstone fields for this gauge symmetry. The Goldstone fields for a gauge symmetry *always* have a pure gauge nature.

On the other hand, the presence of the Goldstone fields also indicates that the gauge diffeomorphism symmetry is spontaneously broken and that a counterpart of Higgs effect should occur in the (super)gravity—(super-) p -brane interacting system. This was discussed in detail in [9]. The counterpart of the mass term for the gauge field appears to be just the source term, *i.e.* the energy-momentum tensor (4) in the *r.h.s.* of the Einstein equation (3). However, as it was shown in [9], this is *not a mass term*.

⁵ Here we do not discuss matter fields which may ‘live’ on the p -brane, like worldvolume vector gauge field of Dirichlet branes.

⁶ This was the point of view accepted by Einstein [11], see also [13], although only particles were considered that time.

When a field equation is considered in a curved spacetime, a term linear in the field entering this equation could be either mass term or a contribution from the nonvanishing curvature of the spacetime. The criterium which allows to define the notions of massive and massless (spin)tensor field in the curved spacetime is the number of polarizations. A massless graviton should possess $D(D-3)/2$ physical polarizations (2 for $D=4$).

Thus, the key point is whether in the (super)gravity–(super)brane interacting system the graviton keeps the same number of polarizations as in the case of ‘free’ gravity, even on the brane worldvolume W^{p+1} despite that one uses a part of diffeomorphism invariance to fix the static gauge (16). This is indeed the case [9] because, in accordance with the generalized Einstein–Grommer theorem [12, 21, 6], the Einstein equation (3) with a singular energy–momentum tensor (4) in the *r.h.s.* produces the p -brane equations of motion (5) as its self-consistency conditions. In the static gauge (16) this brane equations becomes the conditions for the gravitation field on W^{p+1} [9] and play there the role of the gauge fixing conditions that were lost due to the loss of the diffeomorphism symmetry used to fix (locally) the static gauge (16).

This can be understood from the perspective of the above discussion about absence of local degrees of freedom of a (super)brane in the presence of dynamical (super)gravity. Indeed, in the usual Higgs effect, the Goldstone *degrees of freedom* do exist and, after fixing the ‘unitary gauge’ removing the Goldstone *fields*, their degrees of freedom reappear as additional polarizations of the gauge field. These make the gauge field massive, as also reflected by the appearance of the mass term in the gauge field equations. In our case, as a brane does not carry any local degrees of freedom in the presence of dynamical gravity, no degrees of freedom are available to reappear as an additional polarization of graviton when the static gauge (16) is fixed. Hence the graviton keeps the same number of polarizations as in the absence of the brane and, thus, remains massless.

To conclude, let us note that, in general, when diffeomorphism invariance is spontaneously broken, a graviton might get a mass *provided* the Stückelberg or Goldstone *degrees of freedom* are present and, thus, might appear as additional graviton polarizations. The studies of the massive graviton in AdS space [23, 24] refer just to the possibility of introducing the (vector) Stückelberg degrees of freedom in a selfconsistent manner. These might appear, *e.g.* as a bound state of two fields in a free CFT interacting with dynamical gravity on AdS space (but not on the Minkowski space) [24]. In contrast, what has been shown in [9] is that ‘material’ branes, *i.e.* the branes described by a diffeomorphism invariant action, although provide Goldstone *fields* indicating the spontaneous breaking of the gauge diffeomorphism symmetry, these *do not* carry any *degrees of freedom* in the presence of dynamical gravity (see footnote 5). This is the reason why the graviton cannot acquire additional polarizations and cannot get mass in this case. It would be interesting to analyze in this perspective the ‘locally localized gravity’ model of Ref. [25], where a region of AdS_5 space is restricted by two AdS_4 ‘hypersurfaces’.

Acknowledgments. This work has been partially supported by the research grants BFM2002-03681, from the Ministerio de Ciencia y Tecnología de España and EU FEDER funds, #383 from Ukrainian FFR, N 2000-254 from INTAS.

Appendix: Why the cosmological constant cannot be treated as a mass term.

A good example to illustrate the above discussion on the relation of graviton masslessness with diffeomorphism invariance is provided by the Einstein equations with a cosmological constant, $\mathcal{G}_{\mu\nu}(g) = \Lambda g_{\mu\nu}$, whose vacuum solution is given by anti-de Sitter (*AdS*) or by de Sitter (*dS*) space.

Considering small excitations $h_{\mu\nu}$ over the *AdS* metric $g_{\mu\nu}^{AdS}$, $g_{\mu\nu} = g_{\mu\nu}^{AdS} + h_{\mu\nu}$, one finds that the linearized Einstein equation $\mathcal{G}_{\mu\nu}^{AdS}(h) = \Lambda h_{\mu\nu}$ contains a ‘mass-like’ term, *i.e.* a term linear in field $\Lambda h_{\mu\nu}$. However, this term might be not only a mass term but also a contribution from the nontrivial curvature of the *AdS* space. This is indeed the case *as a model possessing full diffeomorphism invariance (and not involving Stückelberg degrees of freedom) is considered.*

To introduce the notion of a massless field in curved space one needs to have a smooth flat spacetime limit. This implies, in particular, that the definition of a massless field should produce the same number of polarizations in flat and curved spacetime.

Now, as the linearized Einstein equations over *AdS* spacetime, $\mathcal{G}_{\mu\nu}^{AdS}(h) = \Lambda h_{\mu\nu}$, possess a linearized diffeomorphism symmetry, one may restrict $h_{\mu\nu}$ by the same number of gauge fixed conditions as in flat spacetime (where one assumes $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and considers the linearized equation $\mathcal{G}_{\mu\nu}^\eta(h) = 0$). This implies that the graviton in *AdS* space has the same number of polarization as in the flat spacetime. Hence the graviton in *AdS* space is massless and the cosmological constant cannot be treated as a mass term.

One might also think of the case of small but nonvanishing cosmological constant Λ : $\Lambda \rightarrow 0$, but $\Lambda \neq 0$ (*cf.* the discussion in [9]), where one may consider decomposition over the flat spacetime, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. Here if in the linear approximation for the weak field $h_{\mu\nu}$ one were to find a term proportional to the field $h_{\mu\nu}$ with a constant coefficient, this could only be the mass term. The first impression might be that $\Lambda h_{\mu\nu}$, which appears in the decomposition of the Einstein equation $\mathcal{G}_{\mu\nu}(\eta) + \mathcal{G}_{\mu\nu}^\eta(h) + \mathcal{O}(hh) = \Lambda \eta_{\mu\nu} + \Lambda h_{\mu\nu}$ is just such a mass term. However, a more careful analysis shows that this is not the case (*cf.* [9]). The flat metric does not solve the Einstein equation with cosmological constant, $\mathcal{G}_{\mu\nu}(g) = \Lambda g_{\mu\nu}$, but rather solves ‘free’ Einstein equation, $\mathcal{G}_{\mu\nu}(\eta) = 0$. As a result, the decomposition of the Einstein equation reads

$$\mathcal{G}_{\mu\nu}^\eta(h) + \mathcal{O}(hh) = \Lambda \eta_{\mu\nu} + \Lambda h_{\mu\nu} . \quad (27)$$

Before turning to the linear approximation in the weak field h , which is needed to search for possible mass term, one has to be convinced that the zero-order approximation is selfconsistent. The *l.h.s.* of Eq. (27) does not contain a zero-order term at all. However, if one considers $\Lambda h_{\mu\nu}$ as a first order term in the weak field $h_{\mu\nu}$, then one has to conclude that $\Lambda \eta_{\mu\nu}$ is the term of zero order; in

this setup the zero order approximation for the equation (27) reads $0 = \Lambda \eta_{\mu\nu}$ and implies vanishing cosmological constant $\Lambda = 0$. Thus, to consider the case of nonvanishing but very small cosmological constant, and, at the same time, to make a selfconsistent weak field approximation *over a flat background*, the only possibility is to assume that Λ is of the same order of ‘smallness’ as $h_{\mu\nu}(x)$, $\Lambda \sim h_{\mu\nu}(x)$. [Actually this is natural from the perspective of decomposition over the AdS background, $g_{\mu\nu} = g_{\mu\nu}^{AdS} + h_{\mu\nu}^0$, as for $\Lambda \rightarrow 0$ $g_{\mu\nu}^{AdS} = \eta_{\mu\nu} + h_{\mu\nu}^{AdS}$ with a small $h_{\mu\nu}^{AdS}$; the above $h_{\mu\nu}$ is given by $h_{\mu\nu} = h_{\mu\nu}^{AdS} + h_{\mu\nu}^0$]. In this setup, however, $\Lambda h(x) \sim h(x)h(x)$. Hence, the term $\Lambda h_{\mu\nu}$ can be considered only together with $\mathcal{O}(hh)$ terms in the *l.h.s.* of Eq. (27); it is a term of second order in the weak field approximation. The first order approximation is given by $\mathcal{G}_{\mu\nu}^\eta(h) = \Lambda \eta_{\mu\nu}$ which contains the constant source term rather than a term linear in the weak field. Thus no mass term appears in the selfconsistent linear approximation. [The constant source does not change the number of field theoretical degrees of freedom; see *e.g.* [6]].

This shows in a different way the masslessness of the graviton in *AdS* space, *i.e.*, that the cosmological constant cannot be considered as a mass term.

References

- [1] J. A. de Azcárraga and J. Lukierski, Phys. Lett. B **113**, 170 (1982). *Phys. Rev.* **D28**, 1337 (1983).
- [2] J. A. de Azcárraga, J. P. Gauntlett, J. M. Izquierdo and P. K. Townsend, Phys. Rev. Lett. **63**, 2443 (1989).
- [3] D. Sorokin, *Kappa symmetry and topological charges*, contribution to this volume, *Preprint DFPD 03/TH/46*.
- [4] I. A. Bandos, J. A. de Azcárraga, J. M. Izquierdo and J. Lukierski, Phys. Rev. Lett. **86**, 4451 (2001); I. A. Bandos, J. A. de Azcárraga, M. Picón and O. Varela, hep-th/0307106.
- [5] I. A. Bandos, J. A. de Azcárraga, J. M. Izquierdo and J. Lukierski, Phys. Rev. D **65**, 021901 (2002) [arXiv:hep-th/0104209].
- [6] I.A. Bandos, J.A. de Azcárraga and J.M. Izquierdo, Phys. Rev. **D65**, 105010 (2002) [arXiv:hep-th/0112207]; hep-th/0201067.
- [7] I.A. Bandos, J.A. de Azcárraga, J.M. Izquierdo and J. Lukierski, *Phys. Rev.* **D67** (2003) 065003 [arXiv:hep-th/0207139].
- [8] I.A. Bandos, J.A. de Azcárraga, J.M. Izquierdo and J. Lukierski, hep-th/0211065.
- [9] I.A. Bandos, J.A. de Azcárraga, J.M. Izquierdo and J. Lukierski, *Phys. Rev.* **D68** (2003) 046004 [arXiv:hep-th/0301255].

- [10] C. Chryssomalakos, J.A. de Azcárraga, J.M. Izquierdo and J.C. Pérez Bueno, Nucl. Phys. **B567**, 293 (2000) [arXiv:hep-th/9904137].
- [11] A. Einstein, Annalen der Physik **49**, 769 (1916).
- [12] A. Einstein and J. Grommer, Sitzbungsber. Preuss. Akad. Wiss. Berlin **1**, 2 (1927); see also L. Infeld and J. Plebański, *Motion and Relativity*, Pergamon press, Warszawa 1960, and refs. therein.
- [13] L. Lusanna and M. Pauri, gr-qc/030140, and refs. therein.
- [14] A. Dabholkar, G.W. Gibbons, J.A. Harvey and F. Ruiz Ruiz, Nucl. Phys. **B340**, 33-55 (1990).
- [15] M.J. Duff, R.R. Khuri and J.X. Lu, Phys. Rep. **259**, 213-326 (1995); K.S. Stelle, BPS Branes in Supergravity, hep-th/9803116.
- [16] M.T. Grisaru, P.S. Howe, L. Mezincescu, B. Nilsson and P.K. Townsend, Phys. Lett. **B162**, 116 (1985); J.A. Shapiro and C.C. Taylor, Phys. Rep. **191**, 221-287 (1990) and refs. therein.
- [17] E. Bergshoeff, E. Sezgin and P.K. Townsend, Phys. Lett. **B189**, 75-78 (1987); Ann. Phys. (NY) **185**, 330 (1988).
- [18] W. Siegel, Phys. Lett. **B128**, 397 (1983).
- [19] A. Achúcarro, J.M. Evans, P.K. Townsend and D.L. Wiltshire, Phys. Lett. **B198**, 441 (1987).
- [20] I. A. Bandos and J. M. Isidro, hep-th/0308102.
- [21] M. Gürses and F. Gürsey, Phys. Rev. **D11**, 967 (1975); C. Aragone and S. Deser, Nucl. Phys. **B92**, 327 (1975).
- [22] J. Hughes, J. Liu and J. Polchinski, Phys. Lett. **B180**, 370 (1986); J. Bagger and A. Galperin, Phys. Lett. **B336**, 25 (1994); S. Bellucci, E. Ivanov and S. Krivonos, Phys. Rev. **D66**, 086001 (2002) (hep-th/0206126), and refs. therein.
- [23] M. Porrati, Phys. Rev. D **65**, 044015 (2002); Mod. Phys. Lett. A **18**, 1793 (2003) [arXiv:hep-th/0306253]; M. J. Duff, J. T. Liu and H. Sati, AIP Conf. Proc. **655**, 155 (2003) [arXiv:hep-th/0207003].
- [24] M. Porrati, JHEP **0204**, 058 (2002) [arXiv:hep-th/0112166].
- [25] A. Karch and L. Randall, JHEP **0105**, 008 (2001) [arXiv:hep-th/0011156].